Paper Reference(s)

# 6678

# Edexcel GCE

## Mechanics M2

## Advanced Subsidiary

## Thursday 7 June 2007 – Morning

## Time: 1 hour 30 minutes

| Materials required for examination | Items included with question papers |
| --- | --- |
| Mathematical Formulae (Green) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

### Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g = 9.8$ m s$^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N26115A

**1.** A cyclist and his bicycle have a combined mass of 90 kg. He rides on a straight road up a hill inclined at an angle $\alpha$ to the horizontal, where $\sin \alpha = \frac{1}{21}$. He works at a constant rate of 444 W and cycles up the hill at a constant speed of 6 m $s^{-1}$.

Find the magnitude of the resistance to motion from non-gravitational forces as he cycles up the hill.

**(4)**

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**2.** A particle $P$ of mass 0.5 kg moves under the action of a single force $\mathbf{F}$ newtons. At time $t$ seconds, the velocity $\mathbf{v}$ m $s^{-1}$ of $P$ is given by

$$\mathbf{v} = 3t^2\mathbf{i} + (1 - 4t)\mathbf{j}.$$

Find

(*a*) the acceleration of $P$ at time $t$ seconds,

**(2)**

(*b*) the magnitude of $\mathbf{F}$ when $t = 2$.

**(4)**

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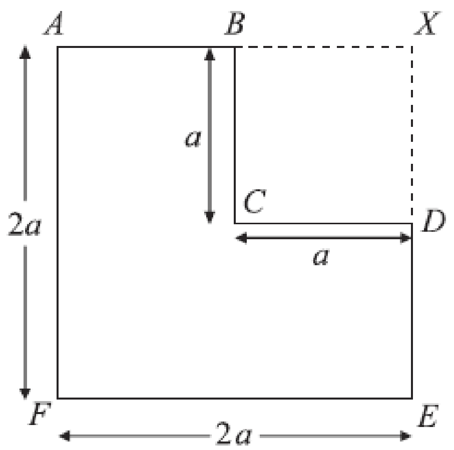
**3.** **Figure 1**




A uniform lamina *ABCDEF* is formed by taking a uniform sheet of card in the form of a square *AXEF*, of side $2a$, and removing the square *BXDC* of side $a$, where *B* and *D* are the mid-points of *AX* and *XE* respectively, as shown in Figure 1.

(*a*) Find the distance of the centre of mass of the lamina from *AF*.

**(4)**

The lamina is freely suspended from *A* and hangs in equilibrium.

(*b*) Find, in degrees to one decimal place, the angle which *AF* makes with the vertical.

**(4)**

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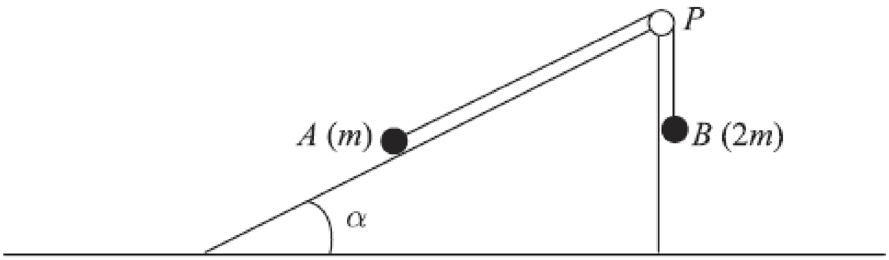
**4.** **Figure 2**




Two particles $A$ and $B$, of mass $m$ and $2m$ respectively, are attached to the ends of a light inextensible string. The particle $A$ lies on a rough plane inclined at an angle $\alpha$ to the horizontal, where $\tan \alpha = \frac{3}{4}$. The string passes over a small light smooth pulley $P$ fixed at the top of the plane. The particle $B$ hangs freely below $P$, as shown in Figure 2. The particles are released from rest with the string taut and the section of the string from $A$ to $P$ parallel to a line of greatest slope of the plane. The coefficient of friction between $A$ and the plane is $\frac{5}{8}$. When each particle has moved a distance $h$, $B$ has not reached the ground and $A$ has not reached $P$.

(*a*) Find an expression for the potential energy lost by the system when each particle has moved a distance $h$.

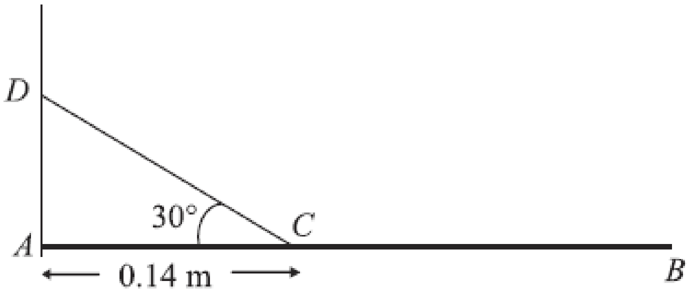
**(2)**

When each particle has moved a distance $h$, they are moving with speed $v$. Using the work-energy principle,

(*b*) find an expression for $v^2$, giving your answer in the form $kgh$, where $k$ is a number.

**(5)**

**5.** **Figure 3**




A uniform beam $AB$ of mass 2 kg is freely hinged at one end $A$ to a vertical wall. The beam is held in equilibrium in a horizontal position by a rope which is attached to a point $C$ on the beam, where $AC = 0.14$ m. The rope is attached to the point $D$ on the wall vertically above $A$, where $\angle ACD = 30°$, as shown in Figure 3. The beam is modelled as a uniform rod and the rope as a light inextensible string. The tension in the rope is 63 N.

Find

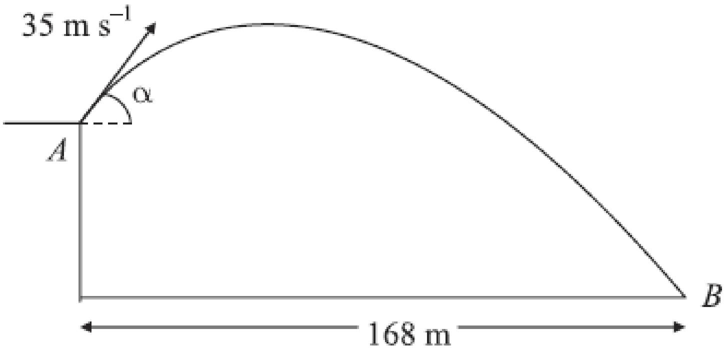
(*a*) the length of $AB$,

**(4)**

(*b*) the magnitude of the resultant reaction of the hinge on the beam at $A$.

**(5)**

**6.** **Figure 4**




A golf ball $P$ is projected with speed 35 m $s^{-1}$ from a point $A$ on a cliff above horizontal ground. The angle of projection is $\alpha$ to the horizontal, where tan $\alpha = \frac{4}{3}$. The ball moves freely under gravity and hits the ground at the point $B$, as shown in Figure 4.

(*a*) Find the greatest height of $P$ above the level of $A$.

**(3)**

The horizontal distance from $A$ to $B$ is 168 m.

(*b*) Find the height of $A$ above the ground.

**(6)**

By considering energy, or otherwise,

(*c*) find the speed of $P$ as it hits the ground at $B$.

**(3)**

**7.** Two small spheres $P$ and $Q$ of equal radius have masses $m$ and $5m$ respectively. They lie on a smooth horizontal table. Sphere $P$ is moving with speed $u$ when it collides directly with sphere $Q$ which is at rest. The coefficient of restitution between the spheres is $e$, where $e > \frac{1}{5}$.

(*a*) (i) Show that the speed of $P$ immediately after the collision is $\frac{u}{6}(5e - 1)$.

(ii) Find an expression for the speed of $Q$ immediately after the collision, giving your answer in the form $\lambda u$, where $\lambda$ is in terms of $e$.

**(6)**

Three small spheres $A$, $B$ and $C$ of equal radius lie at rest in a straight line on a smooth horizontal table, with $B$ between $A$ and $C$. The spheres $A$ and $C$ each have mass $5m$, and the mass of $B$ is $m$. Sphere $B$ is projected towards $C$ with speed $u$. The coefficient of restitution between each pair of spheres is $\frac{4}{5}$.

(*b*) Show that, after $B$ and $C$ have collided, there is a collision between $B$ and $A$.

**(3)**

(*c*) Determine whether, after $B$ and $A$ have collided, there is a further collision between $B$ and $C$.

**(4)**

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**8.** A particle $P$ moves on the $x$-axis. At time $t$ seconds the velocity of $P$ is $v$ m s$^{-1}$ in the direction of $x$ increasing, where $v$ is given by

$$v = \begin{cases} 8t - \frac{3}{2}t^2, & 0 \leq t \leq 4 \\ 16 - 2t, & t > 4. \end{cases}$$

When $t = 0$, $P$ is at the origin $O$.

Find

(*a*) the greatest speed of $P$ in the interval $0 \leq t \leq 4$,

**(4)**

(*b*) the distance of $P$ from $O$ when $t = 4$,

**(3)**

(*c*) the time at which $P$ is instantaneously at rest for $t > 4$,

**(1)**

(*d*) the total distance travelled by $P$ in the first 10 s of its motion.

**(8)**

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**TOTAL FOR PAPER: 75 MARKS**

**END**

June 2007
**6678 Mechanics M2**
**Mark Scheme**

General:
For M marks, correct number of terms, dimensionally correct, all terms that need resolving are resolved.
Omission of *g* from a resolution is an accuracy error, not a method error.
Omission of mass from a resolution is a method error.
Omission of a length from a moments equation is a method error.
Where there is only one method mark for a question or part of a question, this is for a *complete* method.
Omission of units is not (usually) counted as an error.
When resolving, condone sin/cos confusion for M1, but M0 for tan or dividing by sin/cos.

| Question Number | Scheme | Marks |
|---|---|---|
| 1 | Force exerted = 444/6 (= 74 N)<br>$R + 90g \sin \alpha = 44$<br>$\Rightarrow R = \underline{3}$ | B1<br>M1 A1<br>A1<br>(4) |
| | B1 444/6 seen or implied<br>M1 Resolve parallel to the slope for a 3 term equation – condone sign errors and sin/cos confusion<br>A1 All three terms correct – expression as on scheme or exact equivalent<br>A1 32(N) only | |
| 2 .(a) | $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t = 6t\mathbf{i} - 4\mathbf{j}$ | M1 A1 |
| (b) | Using $\mathbf{F} = ½\mathbf{a}$, sub $t = 2$, finding modulus<br>e.g. at $t = 2$, $\mathbf{a} = 12\mathbf{i} - 4\mathbf{j}$<br>$\mathbf{F} = 6\mathbf{i}$ -<br>$\lvert\mathbf{F}\rvert = \sqrt{(6^2 + 2^2)} \approx \underline{6.3}$ | M1, M1, M1<br><br><br>A1(CSO) |
| | M1 Clear attempt to differentiate. Condone **i** or **j** missing.<br>A1 both terms correct (column vectors are OK)<br><br>The 3 method marks can be tackled in any order, but for consistency on epen grid please enter as:<br>M1 **F**=m**a** (their **a**, (correct **a** or following from (a)), not **v**. $\mathbf{F}=\frac{1}{2}\mathbf{a}$).<br>Condone **a** not a vector for this mark.<br>M1 subst t = 2 into candidate's vector **F** or **a** (**a** correct or following from (a), not **v**)<br>M1 Modulus of candidate's **F** or **a** (not **v**)<br>A1 CSO All correct (beware fortuitous answers e.g. from 6t**i**+4**j**)) Accept 6.3, awrt | |

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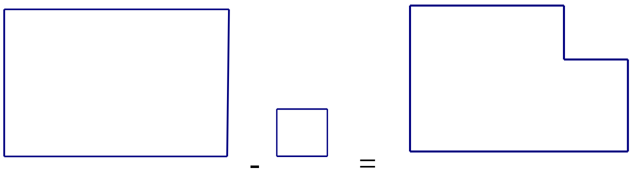
|---|---|---|
| | 6.32, any exact equivalent e.g. $2\sqrt{10}$, $\sqrt{40}$, $\frac{\sqrt{160}}{2}$ | |

| | | |
|---|---|---|
| 3 |  | |
| (a) | M (*AF*) $4a^2.a \;-\; a^2.3a/2 \;=\; 3a^2.\bar{x}$<br>$\bar{x} = \underline{5a/6}$ | M1 A2,1,0<br>A1<br>(4) |
| (b) | Symmetry $\Rightarrow$ $\bar{y} = 5a/6$, or work from the top to get 7a/6<br>$\tan q = \dfrac{5a/6}{2a - 5a/6}$ $\left(\dfrac{\bar{x}}{2a - \bar{y}}\right)$<br>$q \approx \underline{35.5°}$ | B1√<br>M1 A1√<br>A1<br>(4) |
| | M1 Taking moments about AF or a parallel axis, with mass proportional to area. Could be using a difference of two square pieces, as above, but will often use the sum of a rectangle and a square to make the L shape. Need correct number of terms but condone sign errors for M1.<br>A1 A1 All correct<br>A1 A0 At most one error<br>A1 5a/6, ( accept 0.83a or better )<br><br>*Condone consistent lack of a's for the first three marks.*<br><br>*NB: Treating it as rods rather than as a lamina is M0*<br><br>B1ft $\bar{x} = \bar{y} =$their 5a/6, or $\bar{y}$ =distance from AB = 2a - their 5a/6.<br>Could be implied by the working. Can be awarded for a clear statement of value in (a).<br>M1 Correct triangle identified and use of tan. $\dfrac{2a-5a/6}{5a/6}$ is OK for M1.<br>Several candidates appear to be getting 45□ without identifying a correct angle.<br>This is M0 unless it clearly follows correctly from a previous error.<br>A1ft Tan□ expression correct for their 5a/6 and their $\bar{y}$<br>A1 35.5 (Q asks for 1d.p.)<br><br>NB: Must suspend from point A. Any other point is not a misread. | |

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|---|---|---|
| 4. (a)<br><br>(b) | PE lost = $2mgh - mgh\sin\alpha$ ( = $7mgh/5$ )<br><br>Normal reaction $R = mg\cos\alpha$ ( = $4mg/5$ )<br><br>Work-energy: $\frac{1}{2}mv^2 + \frac{1}{2}.2mv^2 = \frac{7mgh}{5} - \frac{5}{8}.\frac{4mg}{5}.h$<br><br>$\Rightarrow \frac{3}{2}mv^2 = \frac{9mgh}{10} \Rightarrow v^2 = \frac{3}{5}gh$ | M1 A1<br>(2)<br>B1<br><br>M1 A2,1,0<br><br><br>A1<br>(5) |
| | M1 Two term expression for PE lost. Condone sign errors and sin/cos confusion, but must be vertical distance moved for A<br>A1 Both terms correct, sin□ correct, but need not be simplified. Allow 13.72*mh*. Unambiguous statement.<br><br>B1 Normal reaction between A and the plane. Allow when seen in (b) provided it is clearly the normal reaction. Must use cos□ but need not be substituted.<br>M1(NB QUESTION SPECIFIES WORK & ENERGY) substitute into equation of the form<br>PE lost = Work done against friction plus KE gained. Condone sign errors. They ***must include KE of both particles.***<br>A1A1 All three elements correct (including signs)<br>A1A0 Two elements correct, but follow their GPE and □ x their R x *h*.<br>A1 $V^2$ correct (NB *kgh* specified in the Q) | |

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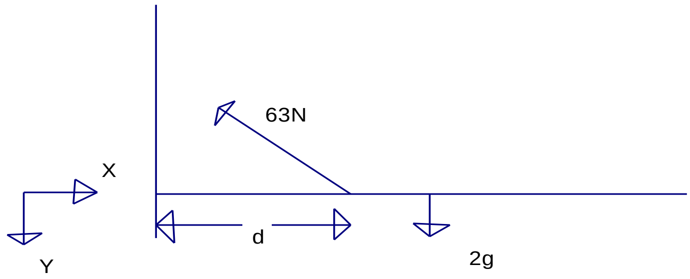
|---|---|---|
| 5.(a) |  | |
| | M($A$) 63 sin 30 . 14 = 2$g$ . $d$<br>Solve: $d$ = 0.225m<br>Hence $AB$ = <u>45 cm</u> | M1 A1 A1<br><br>A1<br>(4) |
| (b) | R(→) $X$ = 63 cos 30 (≈ 54.56)<br><br>R(↑) $Y$ = 63 sin 30 – 2$g$ (≈ 11.9)<br><br>$R = \sqrt{(X^2 + Y^2)}$ ≈ <u>55.8, 55.9 or 56 N</u> | B1<br><br>M1 A1<br><br>M1 A1<br>(5) |
| | M1 Take moments about A. 2 recognisable force x distance terms involving 63 and 2(g).<br>A1 63 N term correct<br>A1 2g term correct.<br>A1 $AB$ = 0.45(m) or 45(cm). No more than 2sf due to use of $g$.<br><br>B1 Horizontal component (Correct expression – no need to evaluate)<br>M1 Resolve vertically – 3 terms needed. Condone sign errors. Could have cos for sin.<br>Alternatively, take moments about B : $0.225 \times 2g = 0.31 \times 63 \sin 30 + 0.45Y$<br>or C : $0.14Y = 0.085 \times 2g$<br>A1 Correct expression (not necessarily evaluated) - direction of Y does not matter.<br>M1 Correct use of Pythagoras<br>A1 55.8(N), 55.9(N) or 56 (N)<br><br>OR For X and Y expressed as Fcos□ and Fsin□ .<br>M1 Square and add the two equations, or find a value for tan □ , and substitute for sin □ or cos □<br><br>A1 As above .<br><br>N.B. Part (b) can be done before part (a). In this case, with the extra information about the resultant force at A, part (a) can be solved by taking moments about any one of several points. M1 in (a) is for a complete method - they must be able to substitute values for all their forces and distances apart from the value they are trying to find.. | |

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|---|---|---|
| 6. (a) | $0 = (35 \sin \alpha)^2 - 2gh$<br>$h = \underline{40 \text{ m}}$ | M1 A1<br>A1 (3) |
| (b) | $x = 168 \Rightarrow 168 = 35 \cos$ □ $. t$ $(\Rightarrow t = 8\text{s})$<br><br>At $t = 8$, $y = 35 \sin \alpha \times t - \frac{1}{2} g t^2$ $(= 28.8 - ½.g.8^2 = -89.6 \text{ m})$<br><br>Hence height of $A$ = <u>89.6 m</u> or 90 m | M1 A1<br><br>M1 A1<br><br>DM1 A1<br>(6) |
| (c) | $½mv^2 = 1/2.m.35^2 + mg.89.6$<br><br>$\Rightarrow v = \underline{54.6 \text{ or } 55 \text{ m s}^{-1}}$ | M1 A1<br>A1<br>(3) |
| | M1 Use of $v^2 = u^2 + 2as$, or possibly a 2 stage method using $v = u + at$ and $s = ut + \frac{1}{2}at^2$<br>A1 Correct expression. Alternatives need a complete method leading to an equation in h only.<br>A1 40(m) No more than 2sf due to use of $g$.<br><br>M1 Use of $x = u\cos$ □ $. t$ to find $t$.<br>A1 $168 = 35 \times their \cos \alpha \times t$<br>M1 Use of $s = ut + \frac{1}{2}at^2$ to find vertical distance for their $t$. (AB or top to B)<br>A1 $y = 35 \sin \alpha \times t - \frac{1}{2} g t^2$ ($u$,$t$ consistent)<br>DM1 This mark dependent of the previous 2 M marks. Complete method for AB. Eliminate t and solve for s.<br>A1 cso.<br>(NB some candidates will make heavy weather of this, working from A to max height (40m) and then down again to B (129.6m))<br><br>OR : Using $y = x \tan \alpha - \frac{gx^2 \sec^2 \alpha}{2u^2}$<br>M1 formula used (condone sign error)<br>A1 x,u substituted correctly<br>M1 □ terms substituted correctly.<br>A1 fully correct formula<br>M1, A1 as above<br><br>M1 Conservation of energy: change in KE = change in GPE. All terms present. One side correct (follow their h).<br>(will probably work $A$ to $B$, but could work top to $B$).<br>A1 Correct expression (follow their h)<br>A1 54.6 or 55 (m/s)<br>OR: M1 horizontal and vertical components found and combined using Pythagoras | |

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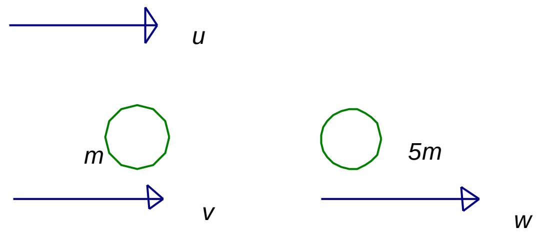
|---|---|---|
| | $v_x = 21$<br>$v_y = 28 - 9.8 \times 8$ (-50.4)<br>A1 $v_x$ and $v_y$ expressions correct (as above). Follow their *h*,*t*.<br>A1 54.6 or 55<br><br>*NB Penalty for inappropriate rounding after use of g only applies once per question.* | |

| Question Number | Scheme | Marks |
| --- | --- | --- |
| 7. |  | |
| (a) | CLM: $mv + 5mw = mu$ | B1 |
| | NLI: $w - v = eu$ | B1 |
| | Solve $v$: $v = \frac{1}{6}(1 - 5e)u$, so speed = $\frac{1}{6}(5e-1)u$ (**NB – answer given on paper**) | M1* A1 |
| | Solve $w$: $w = \frac{1}{6}(1 + e)u$ | M1* A1 |
| | * The M's are dependent on having equations (not necessarily correct) for CLM and NLI | (6) |
| (b) | After $B$ hits $C$, velocity of $B$ = "$v$" = $\frac{1}{6}(1 - 5.\frac{4}{5})u = -\frac{1}{2}u$ | M1 A1 |
| | velocity < 0 ⇒ change of direction ⇒ $B$ hits $A$ | A1 CSO<br>(3) |
| (c) | velocity of $C$ after = $\frac{3}{10}u$ | B1 |
| | When $B$ hits $A$, "$u$" = ½$u$, so velocity of $B$ after = $-\frac{1}{2}(-\frac{1}{2}u) = \frac{1}{4}u$ | B1 |
| | | M1 |
| | Travelling in the same direction but $\frac{1}{4} < \frac{3}{10}$ ⇒ no second collision | A1 CSO<br>(4) |
| | B1 Conservation of momentum – signs consistent with their diagram/between the two equations<br>B1 Impact equation<br>M1 Attempt to eliminate w<br>A1 correct expression for v. Q asks for speed so final answer must be verified positive with reference to e>1/5.<br>**Answer given so watch out for fudges**.<br>M1 Attempt to eliminate v<br>A1 correct expression for w<br><br>M1 Substitute for e in speed or velocity of P to obtain ***v*** in terms of ***u***. Alternatively, can obtain v in terms of w<br>A1 (+/-) u/2 ($v = -\frac{5w}{3}$)<br>A1 CSO Justify direction (and correct conclusion)<br><br>B1 speed of C = value of w = $(\pm)\frac{3u}{10}$ (Must be referred to in (c) to score the B1.) | |

| | | |
|---|---|---|
| | B1 speed of B after second collision $(\pm)\frac{1}{4}u$ or $(\pm)\frac{5}{6}w$<br>M1 Comparing their speed of *B* after 2nd collision with their speed of *C* after first collision.<br>A1 CSO. Correct conclusion . | |
| 8. (a) | $0 \le t \le 4$: $a = 8 - 3t$<br>$a = 0 \Rightarrow t = 8/3$ s<br>$\rightarrow v = 8.\frac{8}{3} - \frac{3}{2}.\left(\frac{8}{3}\right)^2 = \frac{32}{3}$ (m/s)<br>second M1 dependent on the first, and third dependent on the second. | M1<br>DM1<br>DM1 A1<br>(4) |
| (b) | $s = 4t^2 - t^3/2$<br>$t = 4$: $s = 64 - 64/2 = \underline{32\text{ m}}$ | M1<br>M1 A1<br>(3) |
| (c) | $t > 4$: $v = 0 \Rightarrow t = \underline{8\text{ s}}$ | B1 (1) |
| (d) | *Either*<br>$t > 4$ $s = 16t - t^2$ $(+ C)$<br>$t = 4, s = 32 \rightarrow C = -16 \Rightarrow s = 16t - t^2 - 16$<br>$t = 10 \rightarrow s = 44$ m<br>But direction changed, so: $t = 8, s = 48$<br>Hence total dist travelled = 48 + 4 = <u>52 m</u> | M1<br>M1 A1<br>M1 A1<br>M1<br>DM1 A1<br>(8) |
| | *Or (probably accompanied by a sketch?)*<br>t=4 v=8, t=8 v=0, so area under line = $\frac{1}{2}\times(8-4)\times 8$<br>t=8 v=0, t=10 v=-4, so area above line = $\frac{1}{2}\times(10-8)\times 4$<br>total distance = 32(from b) + 16 + 4 = <u>52 m.</u> | M1A1A1<br>M1A1A1<br>M1A1<br>(8) |

| | | |
|---|---|---|
| | Or M1, A1 for t > 4 $\frac{dv}{dt} = -2$, =constant<br>t=4, v=8; t=8, v=0; t=10, v=-4<br>M1, A1 $s = \frac{u+v}{2}t = \frac{32}{2}t$, =16 working for t = 4 to t = 8<br>M1, A1 $s = \frac{u+v}{2}t = \frac{-4}{2}t$, =-4 working for t = 8 to t = 10<br>M1, A1 total = 32+14+4, =52 | |

M1 Differentiate to obtain acceleration
DM1 set acceleration. = 0 and solve for t
DM1 use their t to find the value of v
A1 32/3, 10.7oro better

OR using trial an improvement:
- M1 Iterative method that goes beyond integer values
- M1 Establish maximum occurs for t in an interval no bigger than $2.5<t<3.5$
- M1 Establish maximum occurs for t in an interval no bigger than $2.6<t<2.8$
- A1

Or M1 Find/state the coordinates of both points where the curve cuts the x axis.
- DM1 Find the midpoint of these two values.
- M1A1 as above.

Or M1 Convincing attempt to complete the square:
- DM1 substantially correct $8t-\frac{3t^2}{2}=-\frac{3}{2}(t-\frac{8}{3})^2+\frac{3}{2}\times\frac{64}{9}$
- DM1 Max value = constant term
- A1 CSO

M1 Integrate the correct expression
DM1 Substitute t = 4 to find distance (s=0 when t=0 - condone omission / ignoring of constant of integration)
A1 32(m) only

B1 t = 8 (s) only

M1 Integrate 16-2t
M1 Use t=4, s= their value from (b) to find the value of the constant of integration.
or 32 + integral with a lower limit of 4 (in which case you probably see these two marks
occurring with the next two. First A1 will be for 4 correctly substituted.)
A1 $s = 16t - t^2 - 16$ or equivalent
M1 substitute t = 10
A1 44
M1 Substitute t = 8 (their value from (c))
DM1 Calculate total distance (M mark dependent on the previous M mark.)
A1 52 (m)

OR the candidate who recognizes v = 16 – 2t as a straight line can divide the shape into two triangles:
- M1 distance for t = 4 to t = candidates's 8 = ½ x change in time x change in speed.
- A1 8-4
- A1 8-0
- M1 distance for t = their 8 to t = 10 =½ x change in time x change in speed.
- A1 10-8
- A1 0-(-4)

| | | |
|---|---|---|
| | M1 Total distance = their (b) plus the two triangles (=32 + 16 + 4).<br>A1 52(m)<br><br>NB: This order on epen grid (the A's and M's will not match up.) | |